

INVESTIGATING THE EFFECT OF DIFFERENT RHEOLOGICAL MODELS ON THE BLOOD FLOW IN A CAPILLARY SEGMENT

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INTRODUCTION

The basic elements of human life are carried and sustained by blood. Blood plays a plethora of important roles in the overall health of the body. For centuries, hematologists have made many vital contributions to diagnosing and treating diseases [1].

Numerous recent studies have explored the mathematical and computational models of blood flow. While many articles study blood flow in vessels, few focus on capillaries, and even fewer perform simulations in realistic domains instead of cylindrical approximations. Hence, accurate models of blood rheology are essential for comprehending microcirculation.

Hitherto, there is no universally accepted computational model of the human blood flow. There are many viscosity models used to describe blood flow. Among these, the Carreau model is widely favored due to its superior accuracy in approximating empirically measured blood viscosities. Equally important to choosing a viscous model there is a need to investigate the difference that could arise from using a single-phase model instead of multi-phase blood flow.

In this study, a 3D confocal microscopy image of a capillary segment was analyzed. The segment has a unique structure displaying the geometrical deviations along the segment. Various viscosity models, including both Newtonian and non-Newtonian models, were applied to the steady, laminar, single-phase flow the results are compared.

METHODS

This study investigates different blood flow models using ANSYS Fluent. Initially, the numerical study published by Windes et al. 2016 [2] is used as a reference to investigate the difference between single- and multi-phase flow. Windes et al. 2016 [2] performed simulations of a two-phase blood flow in a cylindrical domain. The diameters and velocities used in those simulations were similar to our case. The flow setting of the 3D cylindrical blood flow included no-slip boundary condition and constant viscosity,

$$\mu_{eff} = \frac{\Delta P}{l} \frac{\pi D^4}{128Q}. \quad (1)$$

The inlet boundary condition of the laminar flow had a fully developed parabolic velocity profile set normal to the inflow velocity. The average inlet velocity was 0.5 mm/s. The simulation results of the five cylindrical cases with different diameters were compared.

A second set of simulations was performed with the 3D confocal microscopy images of capillary segment obtained in collaboration with the Institute of Neurological Sciences and Psychiatry. The .stl segment was processed in SpaceClaim for simulation. The segment had millions of sharp-edged facets that made it difficult to handle, so the geometry was smoothed by merging the facets along the segment. After the merging process, a scaling operation was performed on the segment. This resulted in a segment with a length of around 131 μm and a varying cross-sectional area of 2-9 μm in diameter with an optimal mesh size of around 0.18 μm .

The first model applied to the 3D confocal microscopy images was a non-Newtonian blood model. Several constitutive models, including the Power-Law, Casson, Herschel-Bulkley, and Carreau-Yasuda, account for the shear-rate dependency of blood viscosity. In our simulation, the shear thinning behavior of blood was accounted for by using the Carreau model with properties taken from literature (Table 1).

Table 1: Blood properties of the Carreau model

Density ρ	1050 kg/m ³
Time constant λ	3.313 s
Power-Law Index	0.3568 n
Zero Shear Viscosity μ_0	0.056 kg/m.s
Infinite Shear Viscosity μ_∞	0.0035 kg/m.s

The viscosity of the Carreau model was calculated using the following equation,

$$\mu_{eff} = \mu_\infty + (\mu_0 - \mu_\infty) * [1 + (\lambda \dot{\gamma})^2]^{\frac{n-1}{2}} \quad (2)$$

The second model was the Newtonian model with a constant dynamic viscosity of 0.0035 Pa.s, corresponding to a hematocrit value of $H_t = 0.43$ [3]. This model was used in many studies that investigated the blood flow at a macroscale.

The last investigated model in this study was an empirical equation proposed in 1992 by Pries et al. [4] that relates the relative apparent viscosity of the blood to the tube diameter D and the blood hematocrit H_t ,

$$\mu_{rel} = 1 + (\mu_{0.45} - 1) \frac{(1 - H_t)^{\frac{5}{2}} - 1}{(1 - 0.45)^{\frac{5}{2}} - 1} \quad (3)$$

$$\mu_{0.45} = 220 \exp(-1.3D) + 3.2 - 2.44 \exp(-0.06D^{\frac{4}{5}}) \quad (4)$$

$$C = (0.8 + \exp(-0.075D)) [-1 + (1 + 10^{-11}D^{12})^{-1}] + (1 + 10^{-11}D^{12})^{-1} \quad (5)$$

where $\mu_{0.45}$ is the viscosity that corresponds to the hematocrit value of 0.45. Apart from the viscosity, the domain and the flow settings were identical to the previous models.

The Carreau model and the Newtonian model were first applied to the segment. The vessel walls were assumed to be rigid. A constant inlet velocity of 0.5 mm/s normal to the inlet was assigned. Two outlet boundary conditions—outflow and zero-gauge pressure—were applied for comparison. The fluid density was set to 1050 kg/m³.

The last model applied was the empirical relation where Eq. 3-5 were used to implement different viscosities along the segment. Twelve different simulations were conducted where each section had either a constant viscosity or a linear correlation assigned utilizing the viscosity of the planes at each end of the section.

RESULTS

The pressure drop results of the first set of simulations were very close to those found by Windes et al., 2016 (Table 2). Thus, the choice of single-phase modeling was found to be sufficiently accurate for representing flow in the irregular capillary segment.

Table 2: The pressure drop results of single-phase flow compared with two-phase flow

Diameter r (μm)	Apparent Viscosity (mPa.s)	ΔP Two-Phase Flow [2] (Pa)	ΔP Single-Phase Flow (Pa)
6	1.575	50.4	50.3
7	1.68	35.55	35.7
8	1.785	22.89	22.9
9	1.838	16.1	16.13

Next, we analyzed the results of 4 different models of real capillary geometry. The first model was the Carreau model. The second was the Newtonian model with a constant viscosity of 0.0035 Pa.s. The last two simulations used the empirical relation with a hematocrit of 0.40 and 0.45 respectively.

Finally, we compared the four simulations' pressure (Fig. 1), velocity (Fig. 2), and vorticity findings. Although there is no noticeable difference in the streamlines, the empirical relation with H_t of 0.45 has a little bit higher velocity compared to the other models. The pressure contour, on the other hand, shows a great difference between the models.

DISCUSSION

In this study, the pressure drop was the foremost result, which is expected due to the changing viscosity. Pressure drops ranged from 1.42 kPa for the Newtonian model to 1.70 kPa for the empirical model with a high hematocrit value, while velocity and vorticity remained consistent across models. The results of the Carreau model were closer to those of the empirical relation when compared to the Newtonian

model. Increases in hematocrit or inlet velocity significantly affected pressure drop. It was also noticed that the last model is highly dependent on the cross-sectional area of the geometry; any change in the dimensions has a significant impact on the results. For instance, a 2–3 μm increase in the average diameter of the segment could lower the pressure drop below that of the Newtonian model.

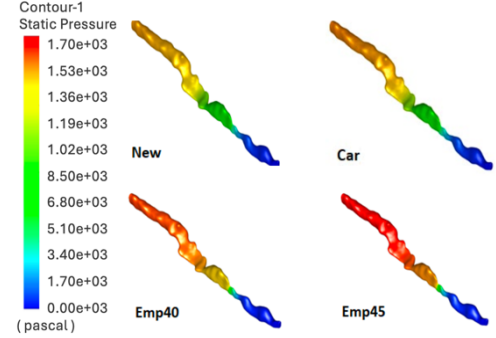


Figure 1: The pressure contour along the capillary segment, (New) Newtonian model, (Car) Carreau model, (Emp40) empirical relation with H_t of 0.40 (Emp45) empirical relation with H_t of 0.45.

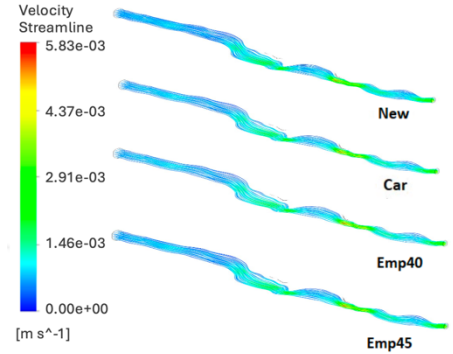


Figure 2: The velocity streamlines along the capillary segment, (New) Newtonian model, (Car) Carreau model, (Emp40) empirical relation with H_t of 0.40 (Emp45) empirical relation with H_t of 0.45.

The study highlights that pressure gradients are key drivers of plasma and cell velocity in capillaries. Changes in the cross-sectional area due to capillary wall actions (e.g., pericyte contractions or endothelial swelling) significantly impact pressure distribution. Proper rheological models are crucial for accurately simulating these effects, with hematocrit and capillary wall dynamics playing important roles in determining pressure gradients.

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